Is there a quad problem among optical gravitational lenses?

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Abstract. Most of optical gravitational lenses recently discovered in the Sloan Digital Sky Survey Quasar Lens Search (SQLS) have two-images rather than four-images, in marked contrast to radio lenses for which the fraction of four-image lenses (quad fraction) is quite high. We revisit the quad fraction among optical lenses by taking the selection function of the SQLS into account. We find that the current observed quad fraction in the SQLS is indeed lower than, but consistent with, the prediction of our theoretical model. The low quad fraction among optical lenses, together with the high quad fraction among radio lenses, implies that the quasar optical luminosity function has a relatively shallow faint end slope.

1. Introduction

Strongly lensed multiple quasars have been known to provide an unique probe of our universe. In particular, the point-source nature of quasars allows a simple statistical study from image multiplicities: Statistics of the number of multiple images provide constraints on the ellipticity and density profile of lens objects as well as the faint end luminosity function of source quasars [1, 2, 3, 4, 5, 6, 7, 8, 9].

The statistics of image multiplicities have been done mainly using radio lenses. [5] adopted a radio lens sample of the Cosmic Lens All-Sky Survey (CLASS) [10, 11] to show that the fraction of four-image (quadruple) lenses is significantly higher than expected from a standard mass model of elliptical galaxies. [6] showed that the fraction of quadruple lenses in a statistical subsample of the CLASS is marginally consistent with what we expect from the observed galaxy population, but it still requires relatively large galaxy ellipticities.

Recent large-scale optical surveys allow us to conduct complementary statistics using optical gravitational lenses. In particular, a large sample of quasars discovered in the Sloan Digital Sky Survey (SDSS) [12] is quite useful for a strong lens survey: Indeed, the SDSS Quasar Lens Search (SQLS) [13] has already discovered approximately 20 new strongly lensed quasars (see, e.g., [14] and references therein), becoming the largest statistical sample of strongly lensed quasars. Interestingly, the fraction of four-image lenses (quad fraction) in the SQLS appears to be significantly lower than the CLASS. Only a few lenses among ~ 20 new SQLS lensed quasars are quadruple lenses, whereas nearly half of CLASS lenses were four (or more) image systems.

In this paper, we revisit the quad fraction among optical gravitational lenses. We adjust the selection function to that of the SQLS and make a comprehensive prediction of the fraction of quadruple lenses. A particular emphasis is paid to whether the current low quad fraction in the SQLS is consistent with the observed galaxy properties. Throughout the paper we adopt Λ-dominated cosmology with the matter density $\Omega_M = 0.3$ and the cosmological constant $\Omega_\Lambda = 0.7$.

2. Calculation

2.1. Lensing Probabilities

We assume that the mass distribution of galaxies can be approximated by an Singular Isothermal Ellipsiod (SIE). The scaled surface mass density of an SIE is given by

$$\kappa(x, y) = \frac{\theta_{\rm E}\lambda(e)}{2}\left[\frac{1-e}{(1-e)^2x^2+y^2}\right]^{1/2}, \tag{1}$$

where e denotes the ellipticity. The Einstein radius $\theta_{\rm E}$ (for $e = 0$) is related with the galaxy velocity dispersion σ by

$$\theta_{\rm E} = 4\pi\left(\frac{\sigma}{c}\right)^2\frac{D_{\rm ls}}{D_{\rm os}}, \tag{2}$$

with $D_{\rm ls}$ and $D_{\rm os}$ being the angular diameter distance from lens to source and from observer to source, respectively. The normalization factor $\lambda(e)$ basically depends on the shape and viewing angle of galaxies: In this paper we assume that there are equal number of oblate and prolate galaxies and adopt the average of the two normalizations (see [6]). We find that with this normalization the Einstein radii are roughly equal for different ellipticities.

It is expected that the quad fraction is mainly determined by the ellipticity. Although the external shear also produces the quadrupole moment in lens potentials, the effect is expected to be minor. For instance, the standard strength of external shear (median value of < 0.05) can cause notable changes in the quad fraction only for lens galaxies with $e < 0.2$ [5]. Therefore throughout the paper we neglect the external shear.

We solve the lens equation using a public code *lensmodel* [15]. The lensing cross section $\sigma_{\rm lens}$ is computed by summing up source positions that yield multiple images with a weight of $\Phi(L/\mu)/\mu/\Phi(L)$, where $\Phi(L)$ is the luminosity function of source quasars and μ is the magnification factor (see §2.2 for which magnification factor we adopt). Lensing cross sections are derived for double and quadruple lenses separately. We compute the image separation for each event from the maximum separation between any image pairs. In computing the lensing probability, we impose a condition that the lensing galaxy should not be brighter than the source quasar, because the lens system may not be targeted for spectroscopy if the lensing galaxy dominates in the flux. We compute the galaxy luminosity from the velocity dispersion adopting an observed correlation [16]. Then the lensing probability of a source at $z = z_s$ becomes

$$\frac{dp_i}{d\theta} = \int_0^{z_s} dz_l \frac{c\,dt}{dz_l}(1+z_l)^3 \int d\sigma \frac{d\sigma_{{\rm lens},i}}{d\tilde{\theta}} \frac{dn}{d\sigma} \delta(\theta - \tilde{\theta}) \Theta(i_{\rm gal} - i_{\rm qso}), \tag{3}$$

with $dn/d\sigma$ being the velocity function of galaxies. The suffix $i = 2$ or 4 denote the number of images.

In computing the lensing probability, we need to specify the lens galaxy population. Since strong lensing is mostly caused by early-type galaxies, particularly for strong lenses in the SQLS whose image separations are basically larger than $1''$, we only consider early-type galaxies. For the velocity function, we assume that of early-type galaxies derived from the SDSS [17, 18]. More important for the quad fraction is the distribution of ellipticities. We adopt a Gaussian distribution with mean $\bar{e} = 0.3$ and the dispersion $\sigma_e = 0.16$, which is consistent with observed ellipticity distributions of early-type galaxies [19, 20, 21, 22, 23], as a fiducial distribution. However we also vary the mean ellipticity, $\bar{e}$, to see how the quad fraction depends on the ellipticity.

2.2. Quasar Population and Selection Function

The quasar luminosity function is another important element to make an accurate prediction of the quad fraction. As a fiducial luminosity function, we adopt that constrained from the combination of the SDSS and 2dF [24]:

$$\Phi(M_g) = \frac{\Phi_*}{10^{0.4(1-\beta_{\rm h})(M_g - M_g^*)} + 10^{0.4(1-\beta_{\rm l})(M_g - M_g^*)}}, \tag{4}$$

Table 1. A current statistical sample of lensed quasars in the SQLS. $N_{\rm img}$ indicate the number of quasar images.

Name	$N_{\rm img}$	$i_{\rm PSF}$	Ref.
SDSS J0246−0825	2	17.8	[27]
SDSS J0746+4403	2	18.8	[14]
SDSS J0806+2006	2	19.0	[28]
SBS0909+523	2	16.2	[29]
SDSS J0924+0219	4	18.2	[30]
FBQ0951+2635	2	17.3	[31]
SDSS J1001+5027	2	17.3	[32]
SDSS J1021+4913	2	19.0	[33]
PG1115+080	4	16.0	[34]
SDSS J1206+4332	2	18.5	[32]
SDSS J1226−0006	2	18.3	[35]
SDSS J1335+0118	2	17.6	[36]
SDSS J1353+1138	2	16.5	[28]

where a pure luminosity evolution with

$$M_g^*(z) = M_g^*(0) - 2.5(k_1 z + k_2 z^2) \tag{5}$$

is assumed. The parameters are $\beta_{\rm h} = 3.31$, $\beta_{\rm l} = 1.45$, $\Phi_* = 1.83 \times 10^{-6}{\rm Mpc}^{-3}{\rm mag}^{-1}$, $M_g^*(0) = -21.61$, $k_1 = 1.39$, and $k_2 = -0.29$. We convert rest-frame g-band magnitudes to observed i-band magnitudes using K-correction derived in [25].

The selection function of the SQLS was studied in detail in [13]. Since the statistical sample of lensed quasars is constructed from quasars with $i < 19.1$ and at $0.6 < z < 2.2$, we restrict our calculation in this range. The magnification bias is computer assuming the image separation dependent magnification factor (see equation (14) in [13]). At $\theta > 1''$ the completeness is almost unity, but there is a small difference of completeness between double and quad lenses: To take this into account we include completeness $\phi_i(\theta)$ in our calculation. In summary, we compute the numbers of double and quad lenses as

$$n_i = \int_{0.6}^{2.2} dz_s \int_{i<i_{\rm lim}} dM_g \Phi(M_g) \Omega D_{\rm os}^2 \frac{c\,dt}{dz_s}(1+z_s)^3 \int_{1''}^{3''} d\theta \phi_i(\theta) \frac{dp_i}{d\theta}, \tag{6}$$

where $i_{\rm lim} = 19.1$ for the statistical lens sample of the SQLS. We have set the upper limit of the image separation to $3''$ since beyond the image separation the effect of surrounding dark matter becomes significant (see, e.g., [26]). The fraction of quadruple lenses is then computed as

$$p_Q = \frac{n_4}{n_2 + n_4}. \tag{7}$$

2.3. Lensed Quasars in the SQLS

The SQLS has already discovered about 20 new lensed quasars as well as several previously known lensed quasars. Although the statistical sample of lensed quasars

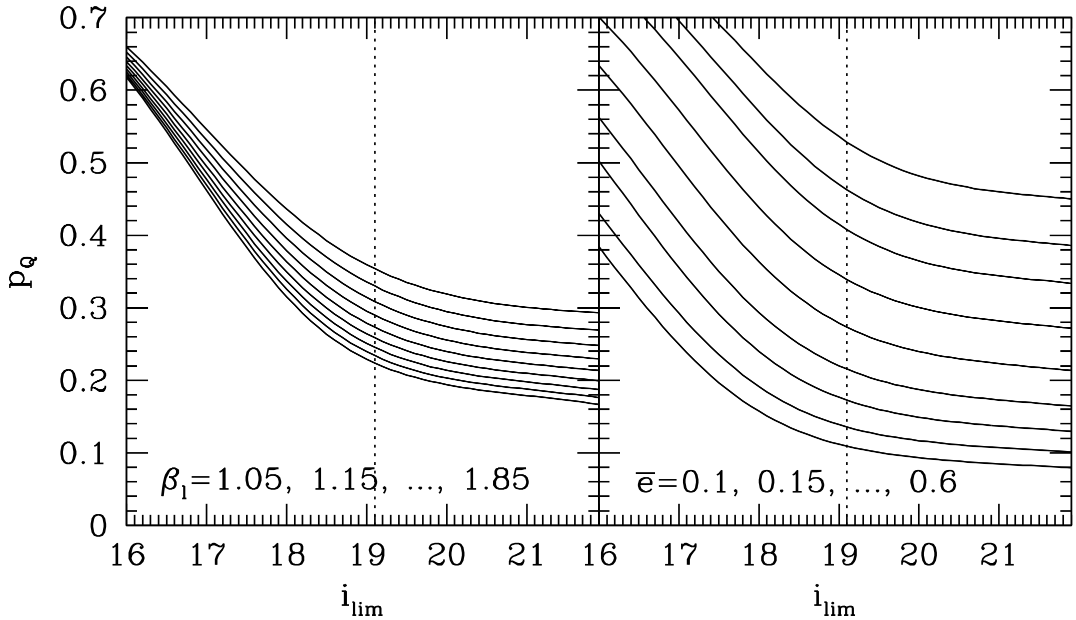

Figure 1. The fraction of quadruple lenses p_Q as a function of i-band limiting magnitude $i_{\rm lim}$. Here we consider lensed quasars with redshifts $0.6 < z < 2.2$, flux ratios $f_i > 10^{-0.5}$, image separations $1'' < \theta < 3''$, and lens galaxies fainter than the quasar components $i_{\rm gal} - i_{\rm qso} > 0$. Dotted line indicate the limiting magnitude of SDSS quasars, $i = 19.1$. Left: From lower to upper solid lines, the faint end luminosity function of quasars $\beta_{\rm l}$ is changed from 1.05 to 1.85. The mean ellipticity $\bar{e}$ is fixed to 0.3. Right: From lower to upper solid lines, the mean ellipticity $\bar{e}$ is changed from 0.1 to 0.5. The slope $\beta_{\rm l}$ is fixed to 1.45.

is still to be finalized, we use these lenses to make a tentative comparison with the theoretical expectation. To make a fair comparison with theory, we select a subsample of lenses by choosing lenses with redshifts $0.6 < z < 2.2$, magnitudes $i < 19.1$, i-band flux ratios (for doubles) $f_i > 10^{-0.5}$, image separations $1'' < \theta < 3''$, and lens galaxies fainter than the quasar components $i_{\rm gal} - i_{\rm qso} > 0$. Currently we have 13 lensed quasars that meet these conditions, which are summarized in Table 1. Among these 13 lenses only two are quadruple lenses, thus the observed quad fraction for the flux limit $i_{\rm lim} = 19.1$ is $p_Q = 2/13 \simeq 0.154$.

3. Result

Before comparing our calculation with the observed quad fraction, we see how it depends on parameters. Among others, the most important parameter is the ellipticity. Another important element that determines the quad fraction is the shape of the quasar luminosity function. In particular the faint end slope $\beta_{\rm l}$ still contains large errors because current large-scale surveys are not deep enough to fully explore the faint end luminosity function. For instance, [37] and [38] adopted the 2dF quasar sample to derive the faint end slopes of $\beta_{\rm l} = 1.58$ and 1.09, respectively. A survey of faint quasars conducted by [39]

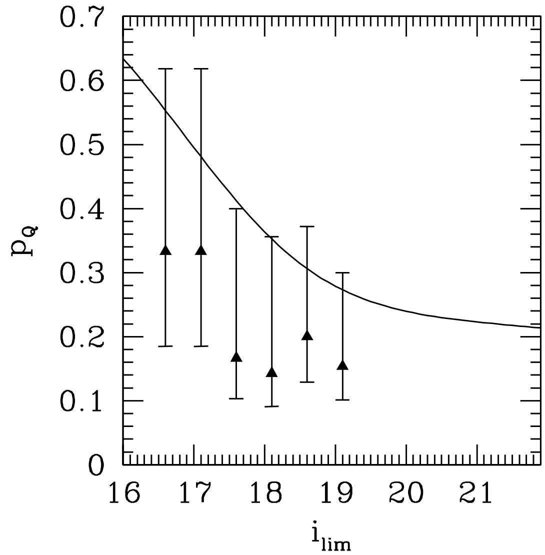

Figure 2. The quad fraction in our fiducial model ($\bar{e} = 0.3$, $\beta_l = 1.45$; shown by a solid line) is compared with observed fractions in the SQLS (filled triangles with errorbars). The errors indicate 68% error estimated assuming the Poisson distribution for the numbers of double and quad lenses. See table 1 for the lens sample we use. Note that the data points are not independent but rather correlated in the sense that lenses used to plot at each $i_{\rm lim}$ are included in computing data points at larger $i_{\rm lim}$ as well.

suggests that the faint end slope could be $\beta_l = 1.25$, shallower than our fiducial value. Other uncertainties, such as cosmological parameters, the velocity function of galaxies, and the number of source quasars, affect the number of double and quad lenses roughly similarly, thus they hardly change the fraction of quad lenses. We find that the effect of changing the prolate/oblate fraction is not large, affecting the quad fraction only by a few percent. Therefore, in figure 1 we plot the quad fraction as a function of the limiting magnitude $i_{\rm lim}$ changing these two important parameters, the mean ellipticity $\bar{e}$ and the faint end slope β_l. First, the quad fraction decreases as the limiting magnitude increases. Larger magnifications of quads than doubles indicate that the quad fraction is a strong function of magnification bias such that larger magnification bias results in larger quad fraction, which explain the decrease of the quad fraction with increasing $i_{\rm lim}$. As expected, the quad fraction is quite sensitive to the ellipticity and the faint end slope of the quasar luminosity function.

Next we compare the quad fraction in our fiducial model with the observed fraction in the SQLS. Figure 2 shows both the theoretical and observed quad fractions as a function of the limiting magnitude. We find that the observed quad fraction is indeed lower than the theoretical prediction. For instance, at $i_{\rm lim} = 19.1$ the quad fraction in our model is $p_Q = 0.273$ that is larger than the observation, $p_Q = 0.154$. However, by taking the large errorbar of the observed fraction due to the small number of lenses, we conclude that the observed quad fraction is consistent with the theoretical expectation.

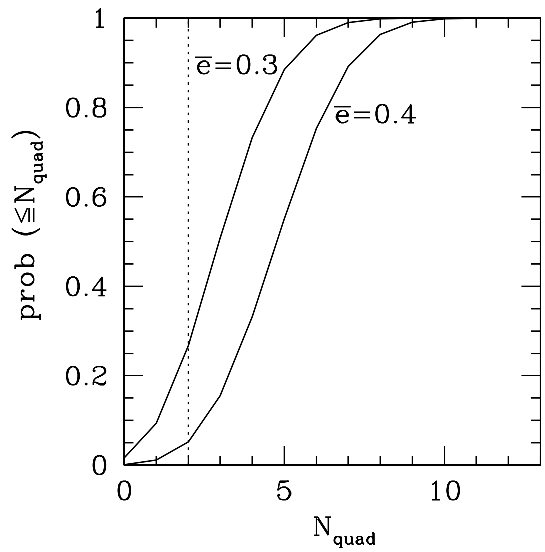

Figure 3. Probability of our model producing quad lenses equal or fewer than N_{quad} in a sample of 13 lenses, computed from our model prediction of the quad fraction for $i_{\mathrm{lim}} = 19.1$, $p_Q = 0.273$. In addition to our fiducial model we also plot the probability for $\bar{e} = 0.4$ ($p_Q = 0.408$) that better reproduces the high quad fraction in the CLASS. The observed number of quads in the SQLS, $N_{\mathrm{quad}} = 2$, is indicated by a vertical dotted line.

In figure 3 we plot the probability that our theoretical model produces $\leq N_{\mathrm{quad}}$ quad lenses in a sample of 13 lenses. Note that in observation there are $N_{\mathrm{quad}} = 2$ quad lenses (see table 1). In our fiducial model the probability is $\simeq 0.27$, which is low but acceptable. On the other hand, if we increase the mean ellipticity to $\bar{e} = 0.4$, which is roughly the best-fit value for the observed quad fraction in the CLASS (see [6]), the probability reduces to $\simeq 0.05$. Therefore with such large-ellipticity model it is difficult to account for the low quad fraction observed in the SQLS.

Finally we check the dependence of the likelihood for $N_{\mathrm{quad}} \leq 2$ in a sample of 13 lenses on the faint end slope β_{l} and the mean ellipticity $\bar{e}$ in figure 4. As expected from figure 1, the probability depends sensitively on these parameters. For instance, by decreasing the faint end slope to $\beta_{\mathrm{l}} = 1.25$, which is preferred by a spectroscopic survey of faint quasars [39], the probability is increased to $\simeq 0.35$. Changing the mean ellipticity to 0.2 enhances the probability to $\simeq 0.61$, making the observed low quad fraction quite reasonable.

4. Summary and Discussion

In this paper, we have studies the fraction of four-image lenses among optical gravitational lenses. We have paid a particular emphasis to whether the low quad fraction observed in the SQLS is consistent with the standard theoretical prediction. In order to make a fair comparison, we have taken account of the selection function and source population in predicting the quad fraction. We find that the observed quad

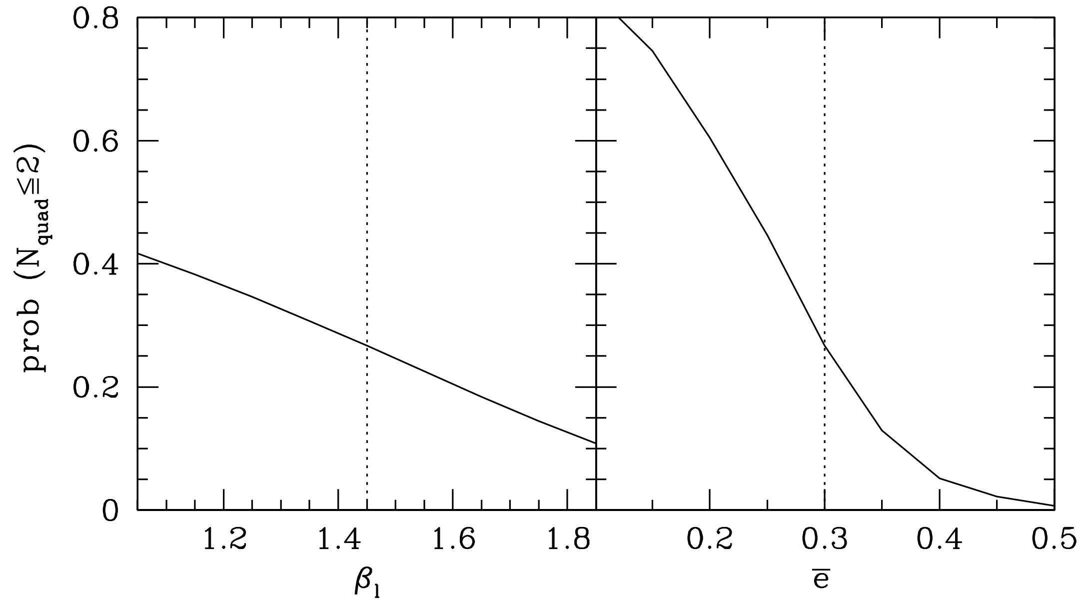

Figure 4. Probability of our model producing quad lenses equal or fewer than the observed case, $N_{\rm quad} = 2$, is plotted as a function of β_l (*left*) or $\bar{e}$ (*right*). The fiducial values are shown by vertical dotted lines.

fraction in the SQLS, $p_Q = 2/13 \simeq 0.154$, is indeed lower than the prediction of our fiducial model, $p_Q = 0.273$, but is consistent given the large Poisson error of the observed quad fraction.

We can lower the expected quad fraction by either making the faint end slope of the quasar luminosity function shallower or decreasing the mean ellipticity of lens galaxies. However, lowering the ellipticity decreases *both* optical and radio quad fractions, therefore such models have difficulty in explaining the high quad fraction among CLASS lenses. For instance, from the CLASS lens sample [6] derived 68% lower limit of the mean ellipticity to 0.28 which is marginally consistent with our fiducial model, $\bar{e} = 0.3$. Therefore, one way to explain both the high quad fraction among radio lenses and low quad fraction among optical lenses is to consider a shallow faint end slope of the quasar optical luminosity function while keeping the mean ellipticity relatively high.

A caveat is that the SQLS is still ongoing and the lens sample is not yet finalized. We should use a final, larger lens sample of the SQLS to draw a more robust conclusion from the quad fraction. The final statistical sample is expected to contain roughly twice the number of lenses we used in this paper, thus the statistical error should be reduced significantly.

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